ME 327: Design and Control of Haptic Systems
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Lecture 16: Teleoperation: Transparency and Stability

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teleoperator transparency

primary teleoperation performance metrics

tracking

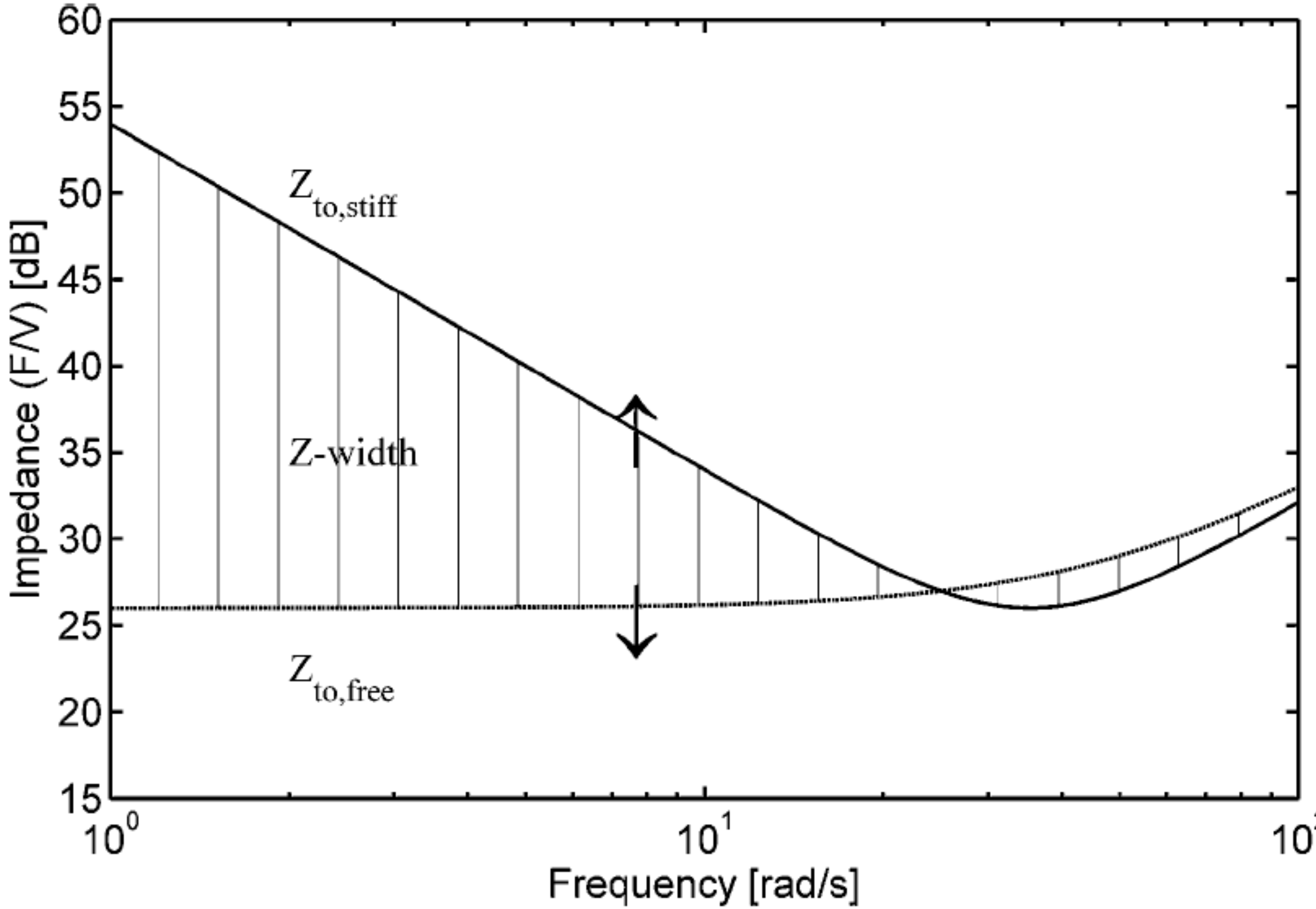
the ability of the follower to follow the master

transparency

(for bilateral teleoperation only)

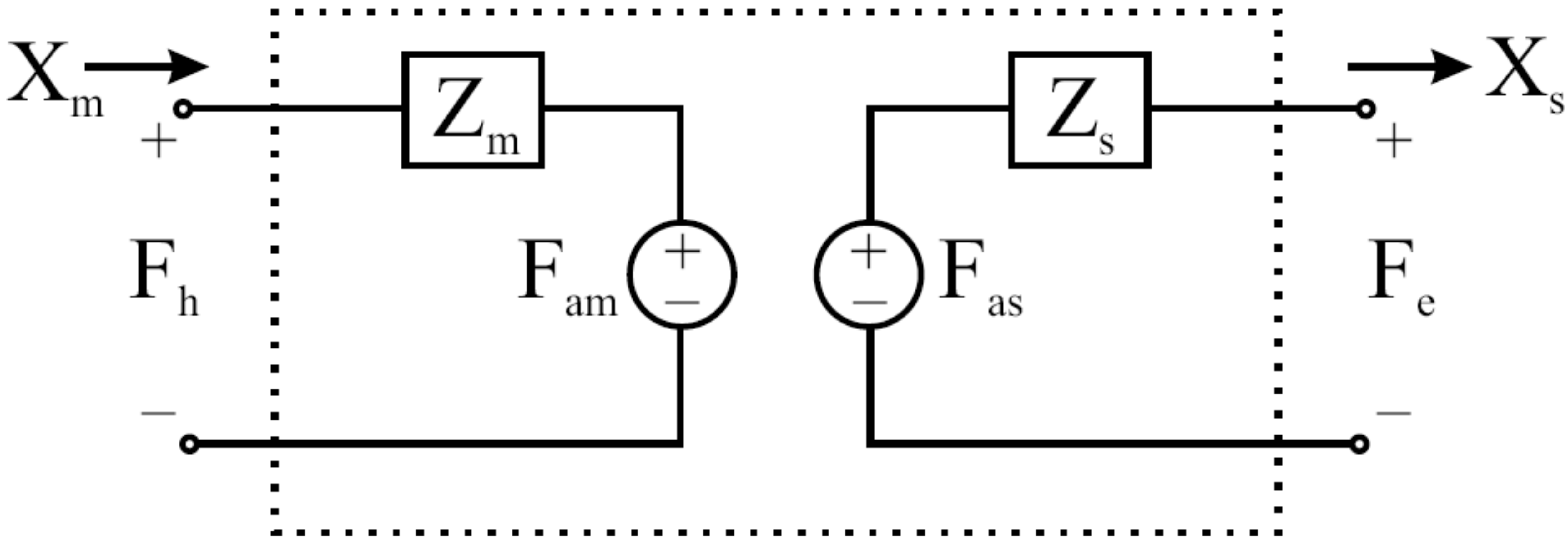
many definitions, but a popular one is whether the mechanical impedance felt by the user is the same as the impedance of the environment

Z-width for teleoperators



Christiansson et al. 2008

system structure



m = master, s = follower (for consistency with literature)

transparency

requirement for perfect transparency
(impedance reflection):

$$Z_e(s) = \frac{F_e(s)}{X_s(s)} \qquad Z_{felt}(s) = \frac{F_h(s)}{X_m(s)}$$

$$Z_e(s) = Z_{felt}(s)$$

a more strict requirement would be:

$$F_h(s) = F_e(s) \quad \text{and} \quad X_m(s) = X_s(s)$$

transparency

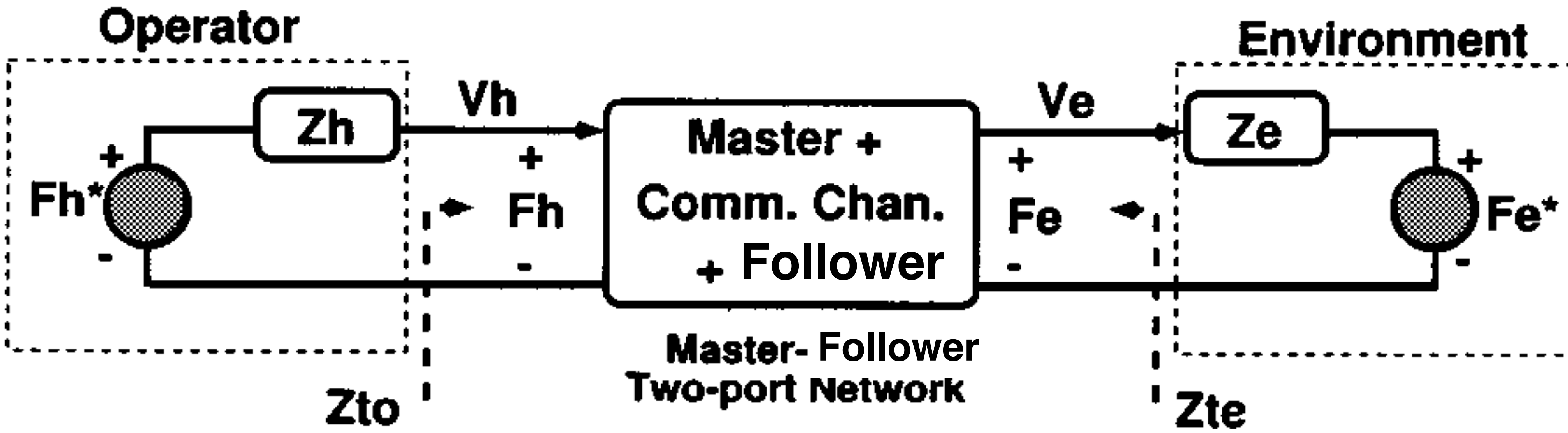
are our three controllers transparent?
you can test each one, using:

$$F_h = X_m Z_m + F_{am}$$

$$F_e = -X_s Z_s + F_{as}$$

The big question: What is required to achieve perfect transparency?

network block diagram



impedance transmitted to the operator

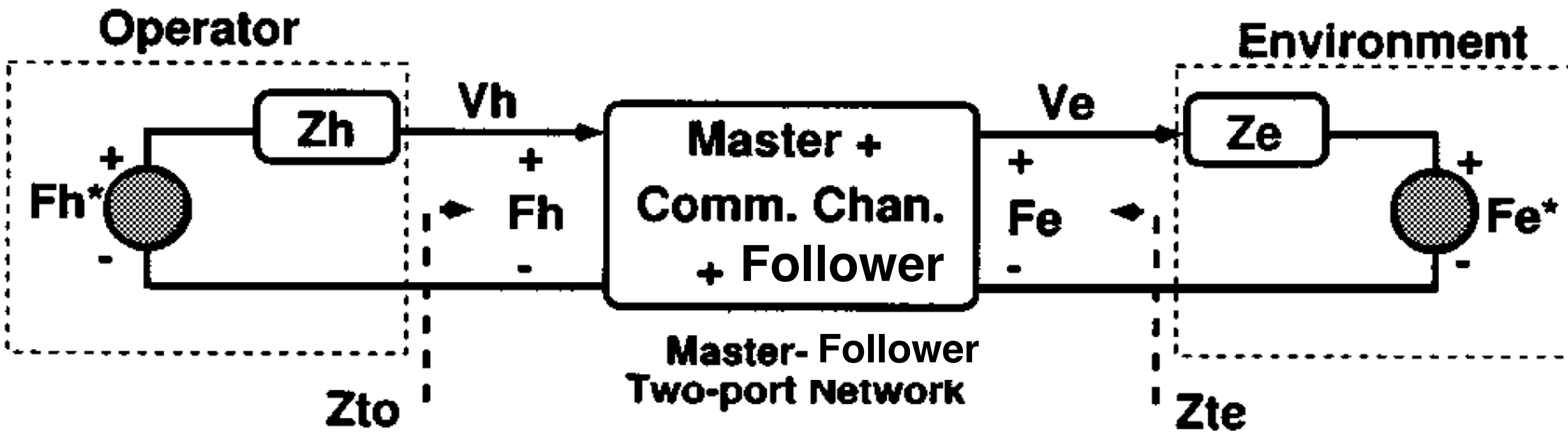
impedance transmitted to the environment

This gives the following relationships, but they are challenging because we don't actually know many of these parameters:

$$F_h = F_h^{\star} - Z_h V_h, \quad F_e = F_e^{\star} + Z_e V_e$$

Hashtrudi-Zaad & Salcudean, 2002

transparency using this notation



we want impedance matching: $Z_{to} = Z_e$ or $Z_{te} = Z_h$

and *perhaps* kinematic correspondence as well: $V_h \equiv V_e$

Hashtrudi-Zaad & Salcudean, 2002

two-port network model

there are four types
here is one (a hybrid model):

$$\begin{bmatrix} F_h \\ -V_e \end{bmatrix} := \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} V_h \\ F_e \end{bmatrix}$$

the hybrid parameters h_{ij} are functions of the master and follower dynamics and their control parameters

the transmitted impedances can be computed:

$$Z_{te} := \frac{F_e}{-V_e}\Big|_{F_h^\star=0} = \frac{h_{11} + Z_h}{(h_{11}h_{22} - h_{12}h_{21}) + h_{22}Z_h}$$

$$Z_{to} := \frac{F_h}{V_h}\Big|_{F_e^\star=0} = \frac{h_{11} + (h_{11}h_{22} - h_{12}h_{21})\,Z_e}{1 + h_{22}Z_e}$$

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but we don't know Z_e and Z_h!

so impedance matching can only be guaranteed when

$$h_{11} = h_{22} = 0$$

$$h_{12}\, h_{21} = -1$$

this will enforce (no matter what Z_e and Z_h are)

$Z_{to} = Z_e$ and $Z_{te} = Z_h$

kinematic correspondence

assuming no position or force scaling, the velocity of the human and the environment are

$$V_h = \frac{\left(F_h^\star - h_{12}F_e^\star\right)\left(h_{22}Z_e + 1\right) + h_{12}h_{22}Z_eF_e^\star}{\left(h_{11} + Z_h\right)\left(h_{22}Z_e + 1\right) - h_{12}h_{21}Z_e}$$

$$V_e = -\frac{\left(h_{11} + Z_h\right)h_{22}F_e^\star + h_{21}\left(F_h^\star - h_{12}F_e^\star\right)}{\left(h_{11} + Z_h\right)\left(h_{22}Z_e + 1\right) - h_{12}h_{21}Z_e}$$

this also requires certain h parameters

$$h_{12}\left(h_{21} + 1\right) = h_{22}\left(h_{11} + Z_h\right)$$

$$\left(h_{21} + 1\right) = -h_{22}Z_e$$

Hashtrudi-Zaad & Salcudean, 2002

total transparency

to enforce perfect impedance reflection *and* kinematic correspondence

$$Z_{to} = Z_e \qquad Z_{te} = Z_h$$

$$V_h \equiv V_e$$

we must have:

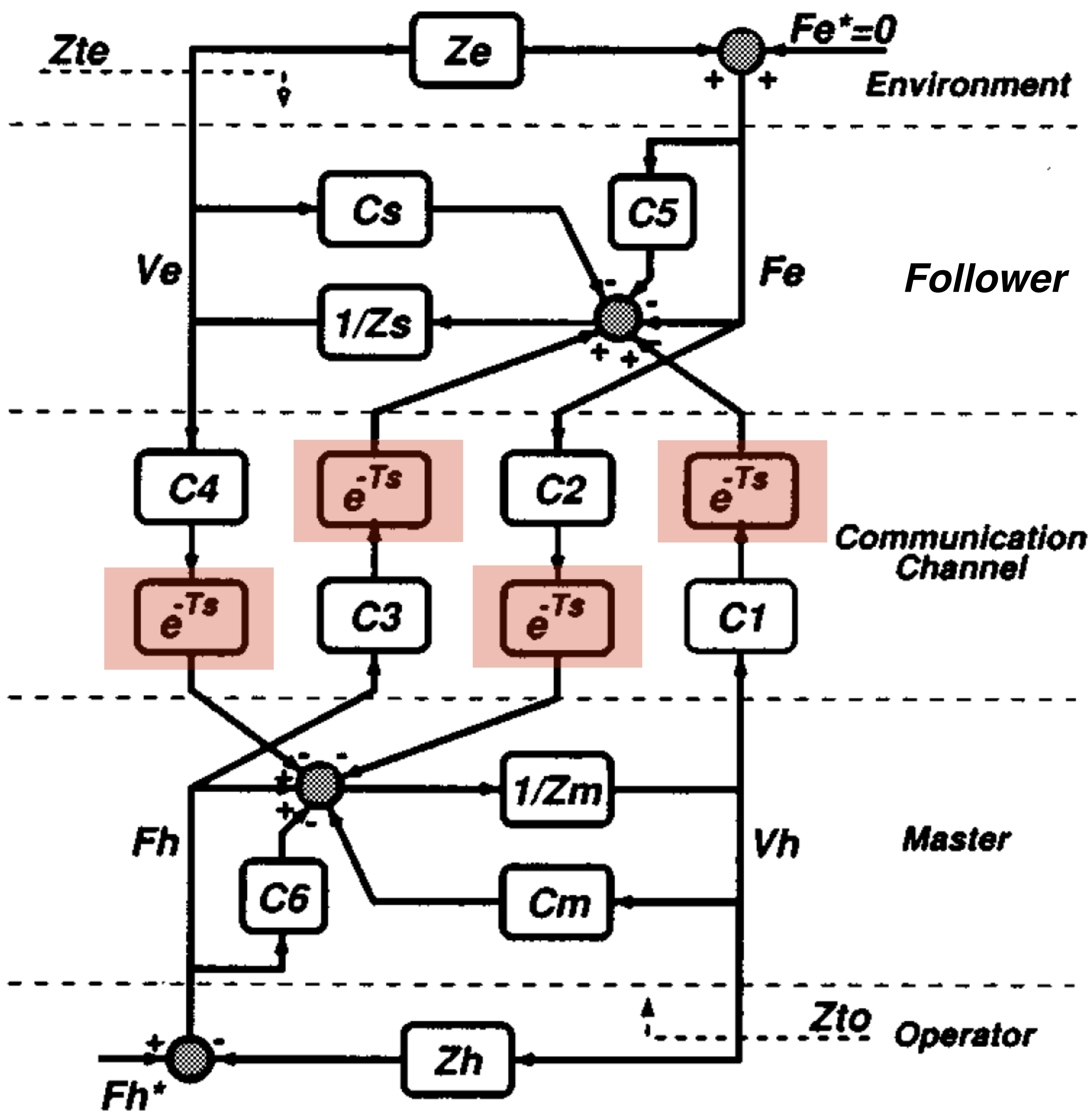
$$h_{11} = h_{22} = 0$$

$$h_{12} = -h_{21} = 1$$

but what *are* these *h*'s??

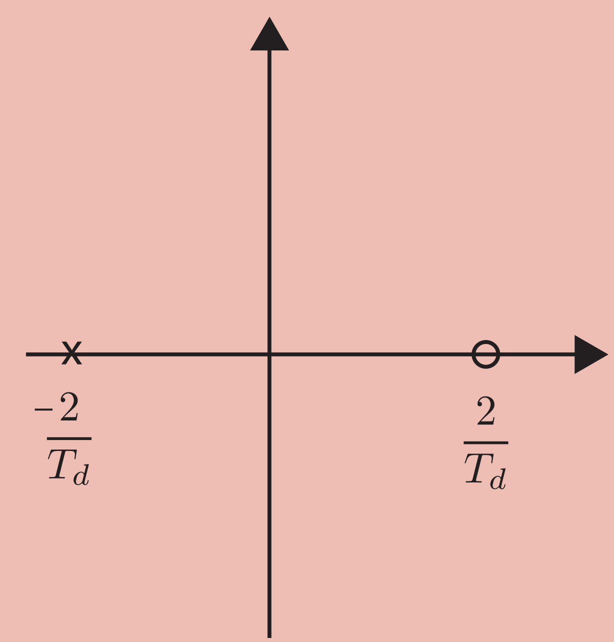
Hashtrudi-Zaad & Salcudean, 2002

a more detailed block diagram



We are ignoring time delay blocks in this class. If you want to include them, use the Padé approximation:

$$e^{-T_d s} \approx \frac{1-\left(\frac{T_d s}{2}\right)}{1+\left(\frac{T_d s}{2}\right)}$$



Hashtrudi-Zaad & Salcudean, 2001

closed-loop equations

master:

$$Z_{cm} V_h + C_4 \boxed{e^{-Ts}} V_e = (1 + C_6) F_h - C_2 \boxed{e^{-Ts}} F_e$$

follower:

$$C_1 \boxed{e^{-Ts}} V_h - Z_{cs} V_e = -C_3 \boxed{e^{-Ts}} F_h + (1 + C_5) F_e$$

where:

$$Z_{cm} = C_m + Z_m \qquad Z_{cs} = C_s + Z_s$$

Hashtrudi-Zaad & Salcudean, 2001

transmitted impedances

$$Z_{te} = \frac{[Z_{cm}Z_{cs} + C_1C_4] + [(1+C_6)Z_{cs} - C_3C_4]\,Z_h}{[(1+C_5)Z_{cm} + C_1C_2] + [(1+C_5)(1+C_6) - C_2C_3]\,Z_h}$$

$$Z_{to} = \frac{[Z_{cm}Z_{cs} + C_1C_4] + [(1+C_5)Z_{cm} + C_1C_2]\,Z_e}{[(1+C_6)Z_{cs} - C_3C_4] + [(1+C_5)(1+C_6) - C_2C_3]\,Z_e}$$

you can match these equations to the equations with the *h*'s to see the relationship between the *h*'s and the control parameters of the system

Hashtrudi-Zaad & Salcudean, 2001

transparency condition

If $C_1 \ldots C_6$ are not functions of Z_h and Z_e

and $(C_2, C_3) \neq (0,0)$

$$C_1 = Z_{cs}$$
$$C_2 = 1 + C_6$$
$$C_3 = 1 + C_5$$
$$C_4 = -Z_{cm}$$

physical interpretation: the master and follower dynamics have to be cancelled out by inverse dynamics and the feedforward forces have to match the net forces exerted by the operator on the environment

Hashtrudi-Zaad & Salcudean, 2001

teleoperator stability

two-port network model

there are four types (here are all of them):

$$\begin{bmatrix} F_h \\ F_e \end{bmatrix} = \mathcal{O}_{\mathcal{Z}} := \mathcal{Z}\mathcal{I}_{\mathcal{Z}} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} V_h \\ -V_e \end{bmatrix}$$

$$\begin{bmatrix} V_h \\ -V_e \end{bmatrix} = \mathcal{O}_{\mathcal{Y}} := \mathcal{Y}\mathcal{I}_{\mathcal{Y}} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} F_h \\ F_e \end{bmatrix}$$

$$\begin{bmatrix} F_h \\ -V_e \end{bmatrix} = \mathcal{O}_{\mathcal{H}} := \mathcal{H}\mathcal{I}_{\mathcal{H}} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} V_h \\ F_e \end{bmatrix}$$

$$\begin{bmatrix} V_h \\ F_e \end{bmatrix} = \mathcal{O}_{\mathcal{G}} := \mathcal{G}\mathcal{I}_{\mathcal{G}} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} F_h \\ -V_e \end{bmatrix}$$

where the *immitance* parameters (called p_{ij} in general) are functions of the master and follower dynamics and their control parameters

stability criteria

an LTI two-port network is absolutely stable if and only if

1. p_{11} and p_{22} have no poles in the right half plane

2. any poles of p_{11} and p_{22} on the imaginary axis are simple (of order one) and have real and positive residues. (The residue is the constant a_{n-1} in the series $f(z) = \sum_{n=-\infty}^{\infty} a_n (z - z_0)^n$)

and…

stability criteria

3. the following inequalities hold:

$$\mathbb{R}\{p_{11}\} \;\; \geq \;\; 0$$

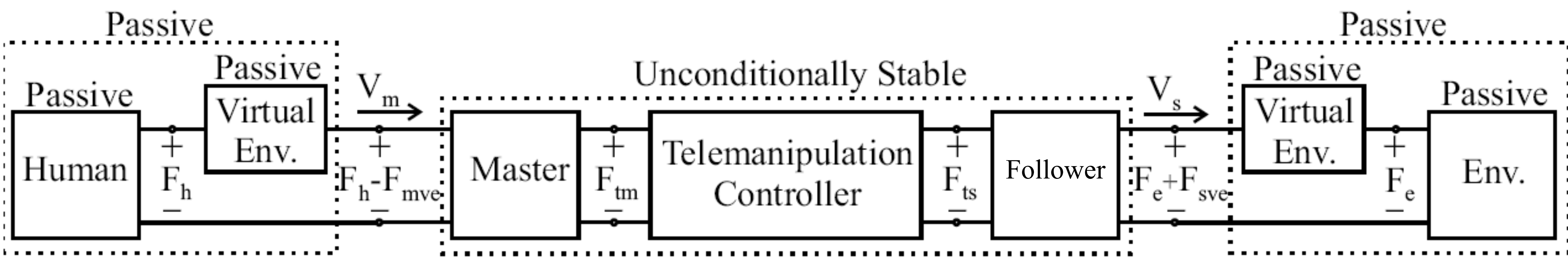
$$\mathbb{R}\{p_{22}\} \;\; \geq \;\; 0$$

$$2\mathbb{R}\{p_{11}\}\mathbb{R}\{p_{22}\} - \mathbb{R}\{p_{12}p_{21}\} - |p_{12}p_{21}| \;\; \geq \;\; 0$$

$$\text{on the } j\omega \text{ axis for all } \omega \geq 0$$

what can you do with this?

you can link up an unconditionally stable system with other passive systems, and the whole thing will be stable



<u>assumptions:</u>

system is LTI

no sample and hold, no ZOH

no time delay or discretization

Abbott Ph.D. Thesis (2005)

robustness

"Although four-channel control architectures can provide stable perfectly transparent systems in theory, stability and performance for these systems are compromised due to the communication-channel delay as well as the operator and environment dynamic uncertainties."

Hashtrudi-Zaad & Salcudean, 2001