ME 327: Design and Control of Haptic Systems
Spring 2020

Lecture 9:
Kinesthetic haptic devices: Dynamics and Control

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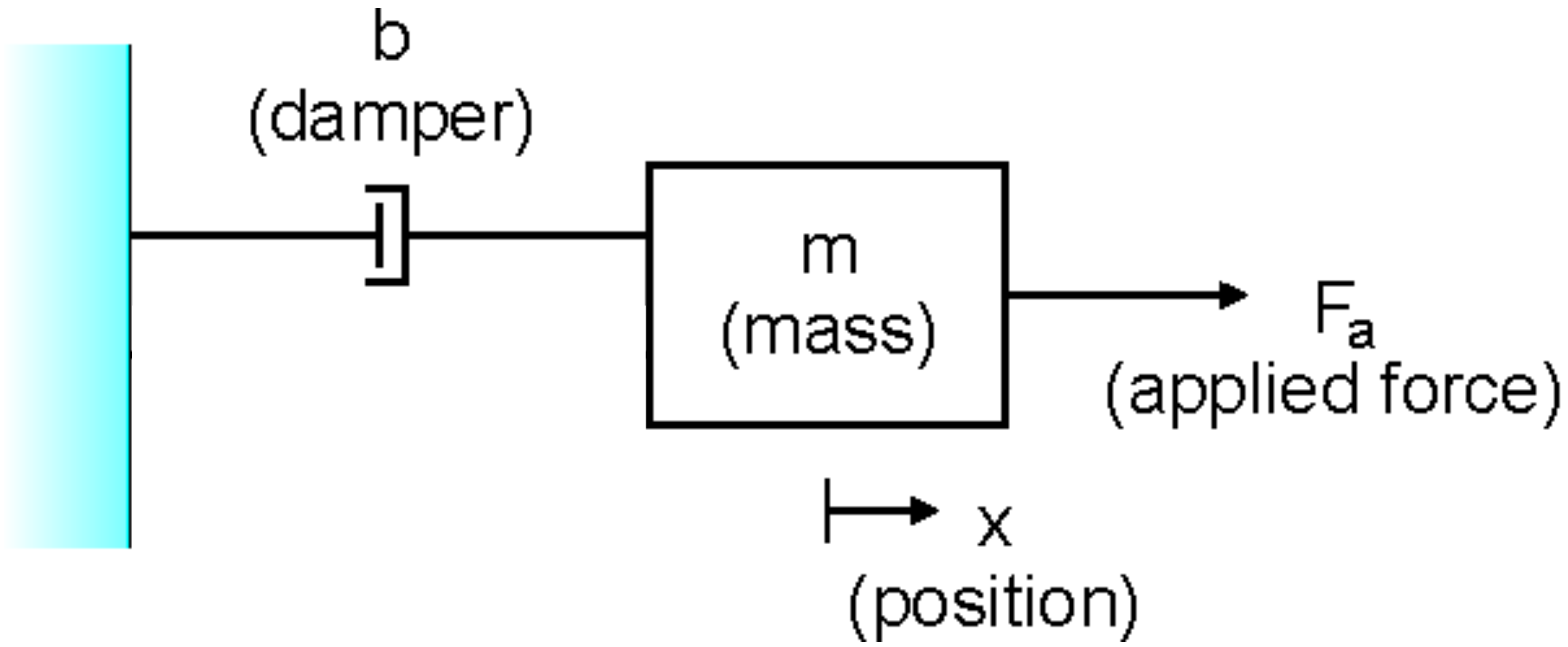
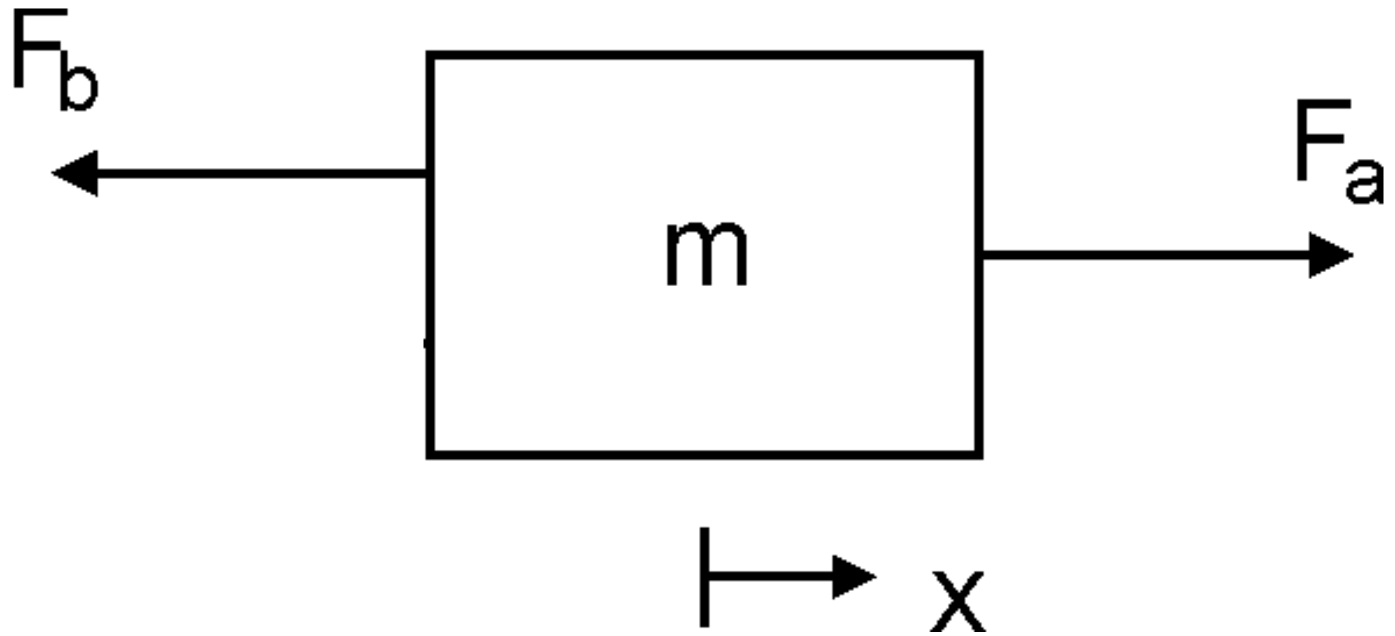
dynamic model

mass-damper model



on the device itself,
there is no spring!

free body diagram



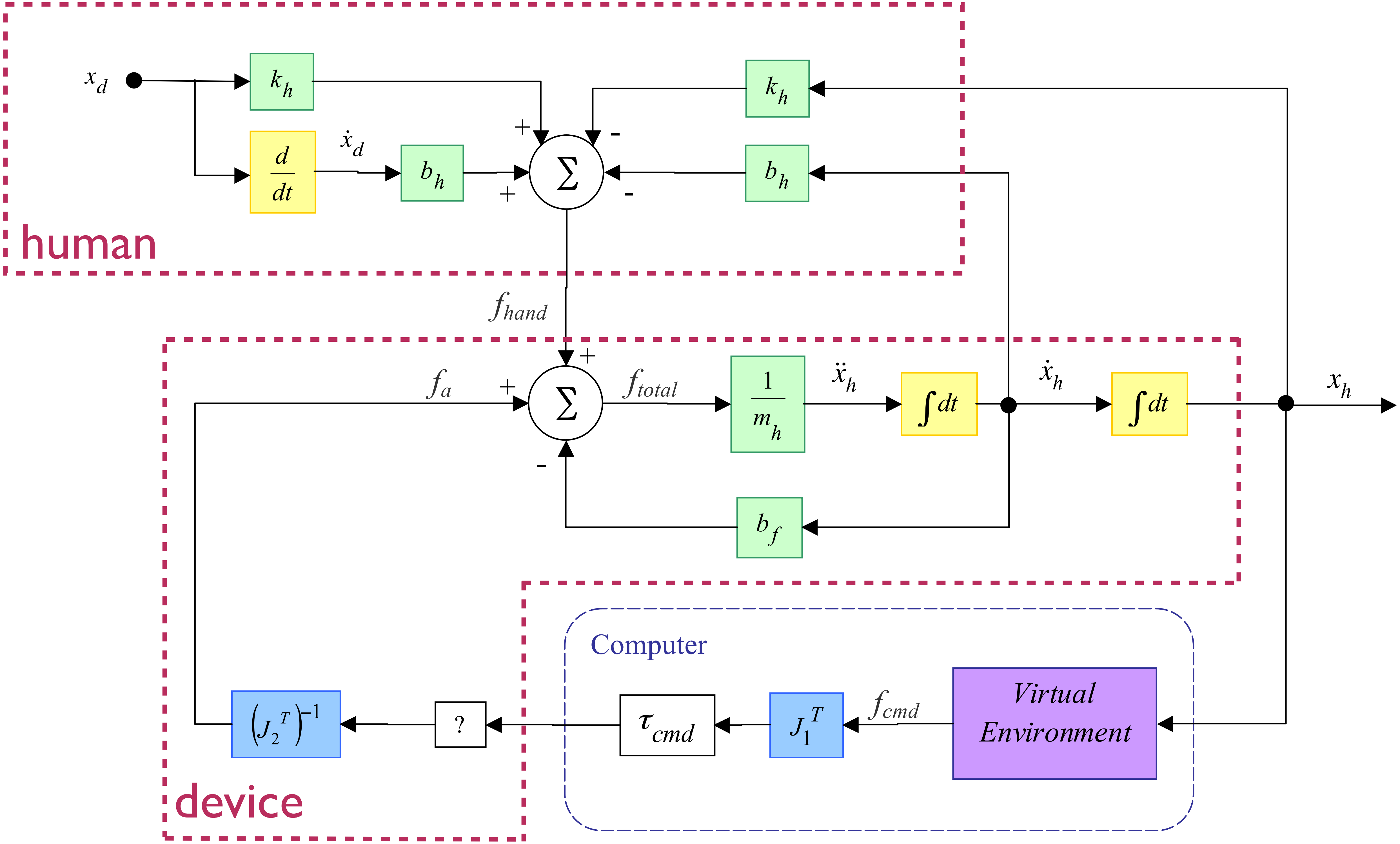
$F_b = b\dot{x}$,

sum forces, equate to inertial term:

$m\ddot{x} = F_a - F_b$

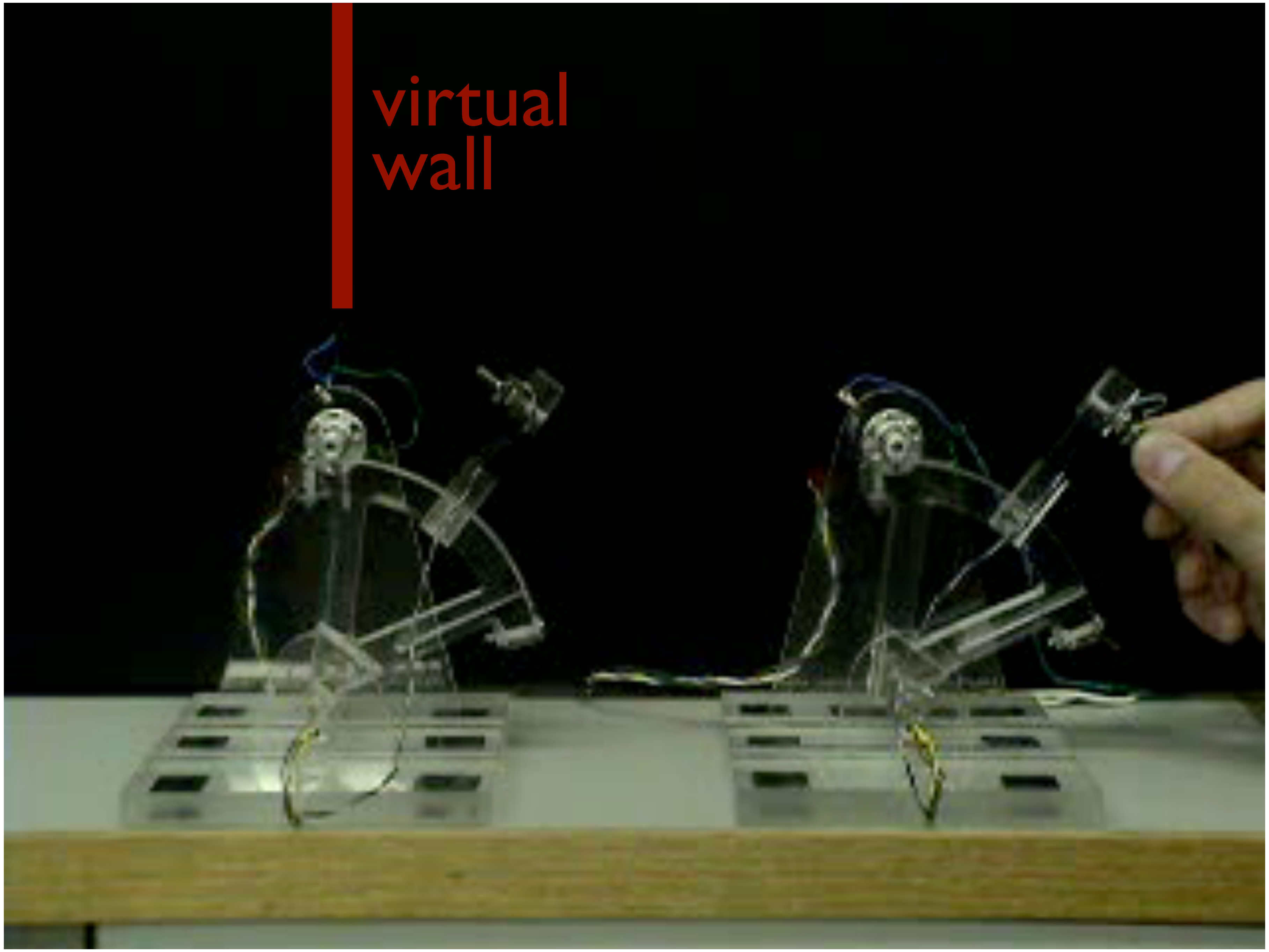
$m\ddot{x} + b\dot{x} = F_a$

system block diagram



important stability concepts

instability / limit cycle oscillation



video credit: Jake Abbott

review stability in the context of the s-plane

common second-order system: $m\ddot{x} + b\dot{x} + kx = f$

take the Laplace transform of both sides:

$$\mathcal{L}[m\ddot{x} + b\dot{x} + kx] = \mathcal{L}[f]$$

$$ms^2X(s) + bsX(s) + kX(s) = F(s)$$

$$(ms^2 + bs + k)X(s) = F(s)$$

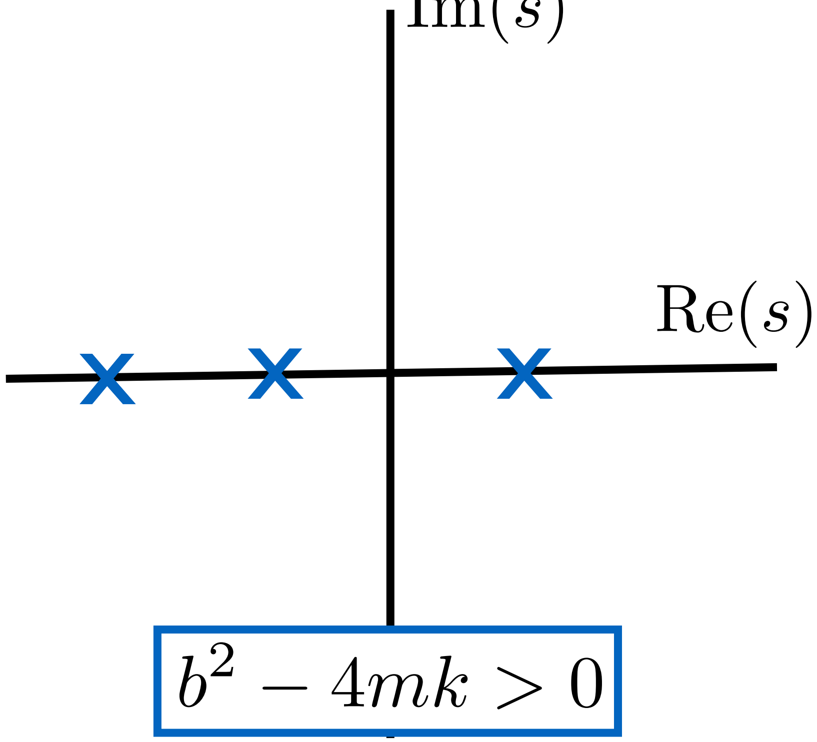
transfer function/characteristic equation: $\dfrac{F(s)}{X(s)} = ms^2 + bs + k$

review stability in the context of the s-plane

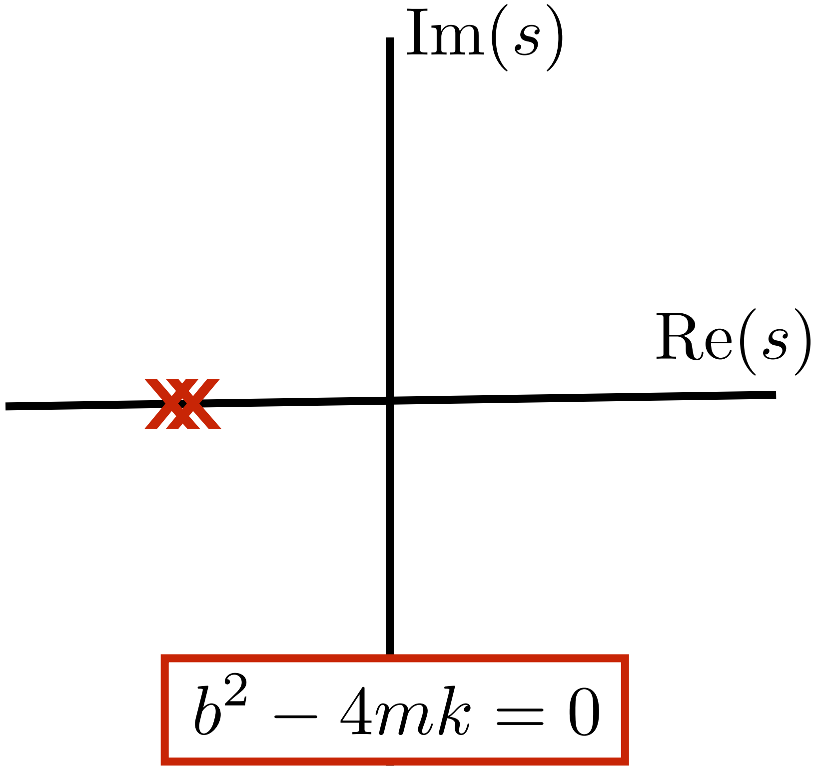
roots of the characteristic equation:

$$s = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

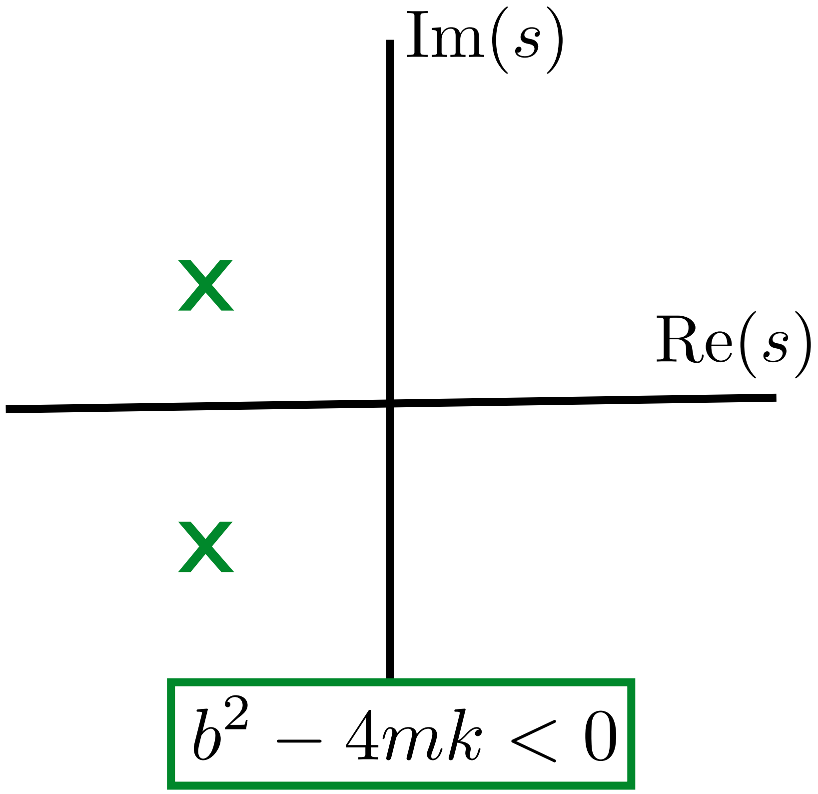
plot roots on the s-plane for any values of $m,\ b$, and k:



$b^2 - 4mk > 0$



$b^2 - 4mk = 0$



$b^2 - 4mk < 0$

why do instabilities occur?

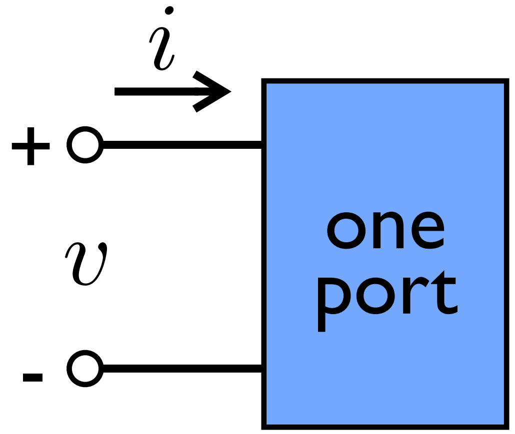
fundamentally, instability has the potential to occur because real-world interactions are only approximated in the virtual world

although these approximation errors are small, their potentially **non-passive** nature can have profound effects, notably:

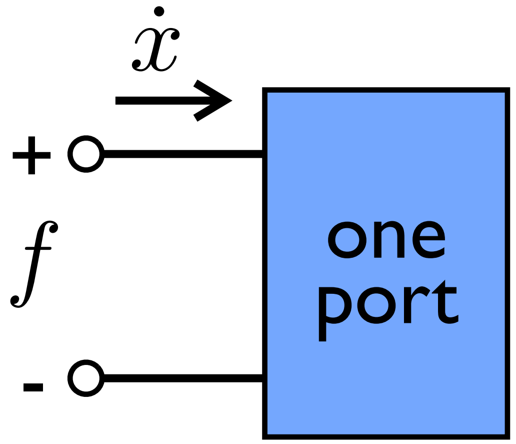
- instability
- limit cycle oscillations (which can be just as bad as instability)

passivity

a useful tool for studying the stability and performance of haptic systems



a one-port is passive if the integral of power extracted over time does not exceed the initial energy stored in the system.



$$\int_0^t f(\tau)\dot{x}(\tau)d\tau \geq 0, \qquad \forall t \geq 0$$

Z-width

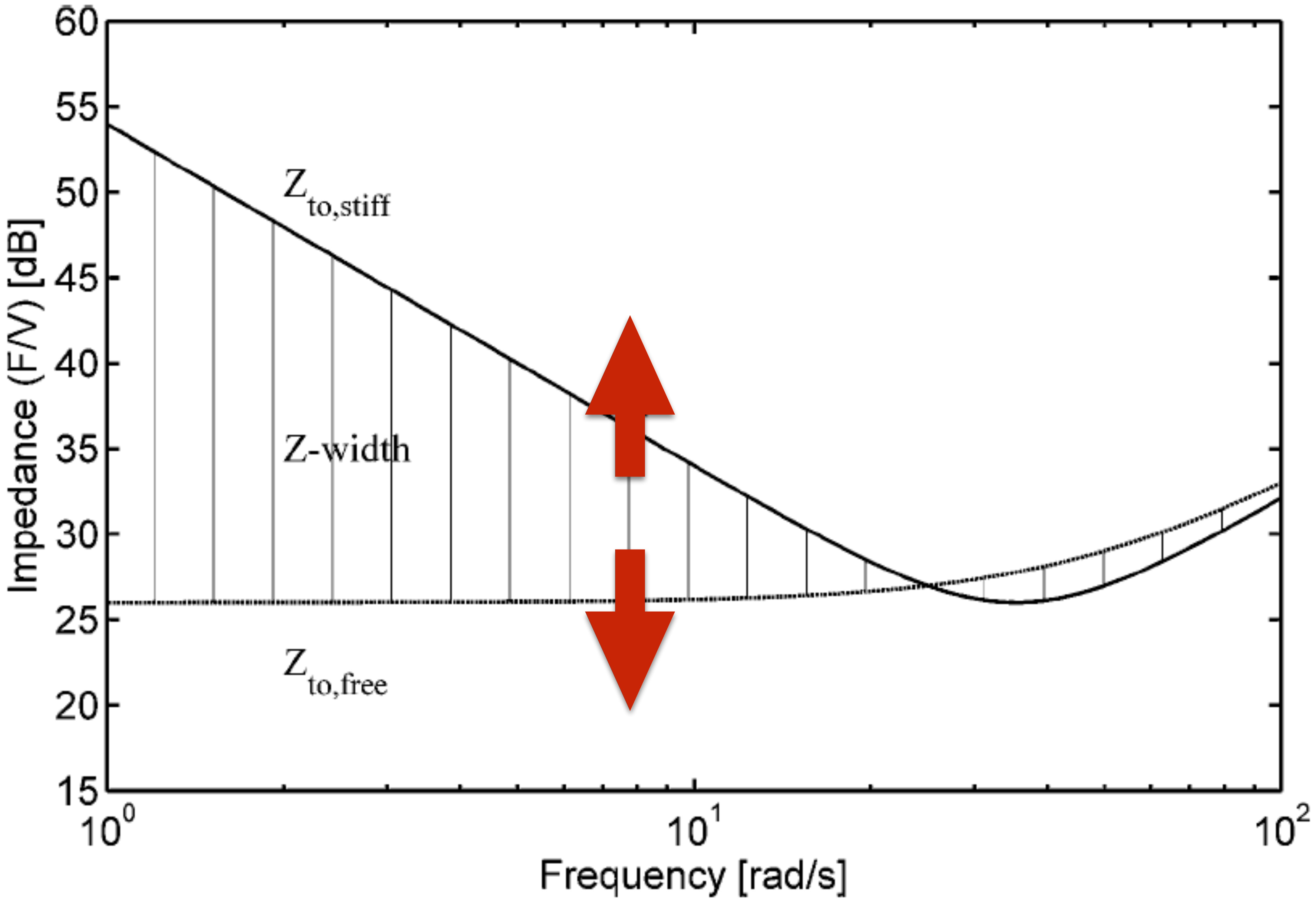
“Z-width” is the dynamic range of impedances that can be rendered with a haptic display while maintaining passivity

we want a large z-width, in particular:

- zero impedance in free space
- large impedance during interactions with highly massive/viscous/stiff objects

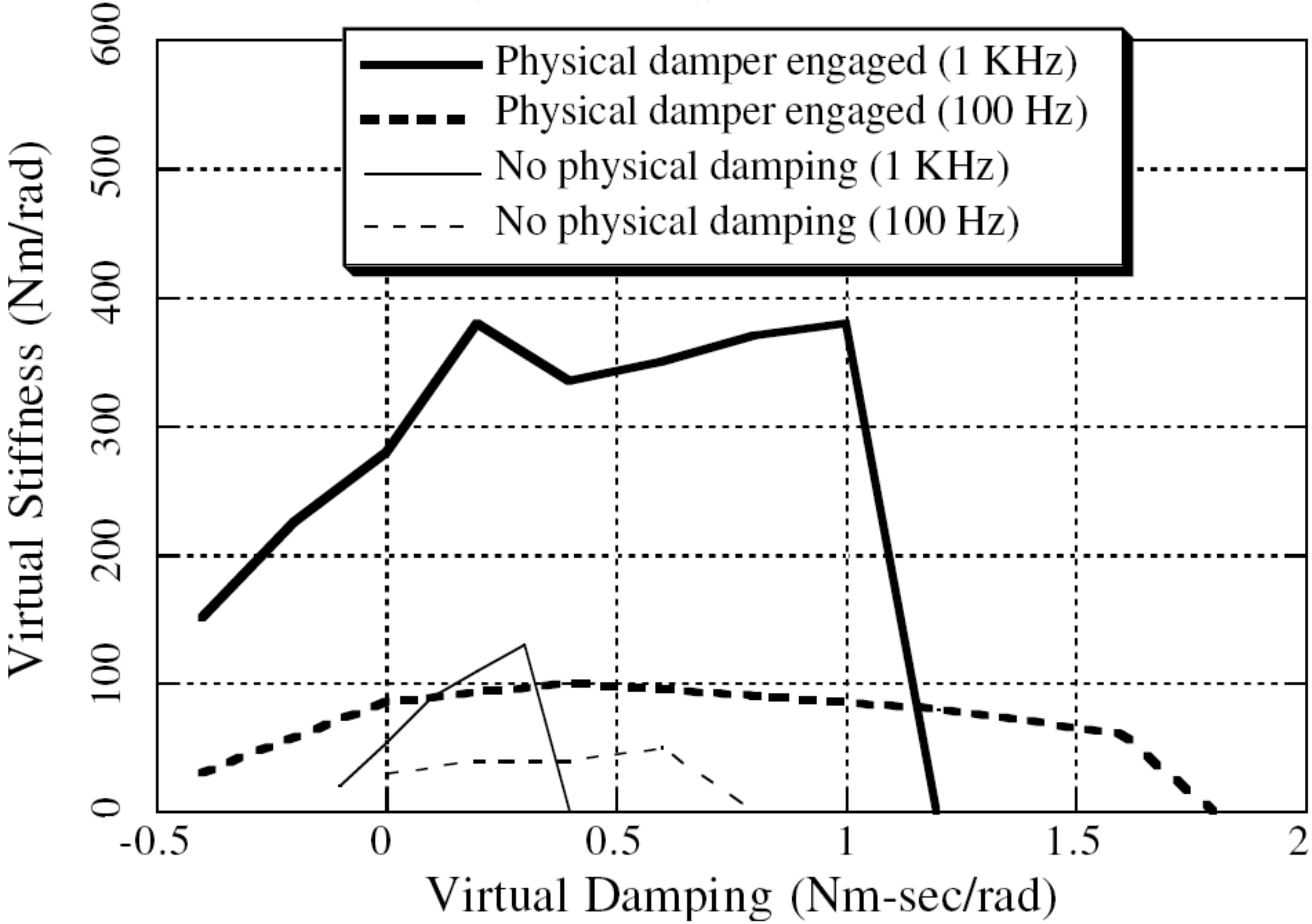
$$\frac{F(s)}{X(s)} = ms^2 + bs + k = Z(s) \qquad F(s) = Z(s)X(s)$$

Z-width



Christiansson et al. 2008

Z-width (experimental)



Colgate and Brown 1994

how do you improve Z-width?

lower bound depends primarily on mechanical design (can be modified through control)

upper bound depends on sensor quantization, sampled data effects, time delay (in teleoperators), and noise (can be modified through control)

in a different category are methods that seek to create a perceptual effect (e.g., event-based rendering)